

While analyzing the NodeIterator algorithm, we want to bound the following expression:

$$\sum_{v \in V} \deg^*(v)^2$$

Consider those nodes for which $\deg^*(v) < t$. Then for that partial sum we have:

$$\begin{aligned} \sum_{v \in V, \deg^*(v) < t} \deg^*(v)^2 &\leq \sum_{v \in V, \deg^*(v) < t} t \deg^*(v) = \\ &= t \sum_{v \in V, \deg^*(v) < t} \deg^*(v) \leq 2mt \end{aligned}$$

Now consider those nodes for which $\deg^*(v) \geq t$. There are at most $2m/t$ such nodes. Note that $\sum_{v \in V, \deg^*(v) \geq t} \deg^*(v)^2$ is upper bounded by the number of triangles between high-degree nodes, and since there are at most $2m/t$ such nodes, we have

$$\sum_{v \in V, \deg^*(v) \geq t} \deg^*(v)^2 \leq (2m/t)^3$$

The total sum is bounded by the sum of the two partial sums we analyzed:

$$\sum_{v \in V} \deg^*(v)^2 \leq (2m/t)^3 + 2mt$$

We can set $t = \sqrt{m}$ to minimize the above expression. Doing so, it becomes bounded by $O(m^{3/2})$