**CME 305: Discrete Mathematics and Algorithms**

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**Midterm Winter 2016**

1. (10 points) Prove that the MAX-CUT problem can be solved in polynomial time, on trees.

2. (15 points) A *kettle* graph on $2n$ nodes is a clique on $n$ nodes, with two arbitrary identified nodes $a$ and $b$. Separate from the clique, there is a path of length $n+2$ between $a$ and $b$. The two ends of the path are $a$ and $b$ and there are $n$ nodes which are not part of the clique on the path.

   (a) (5 points) Show that a kettle graph on $2n$ nodes has cover time $O(n^3)$.

   (b) (10 points) Show that a kettle graph on $2n$ nodes has cover time $\Omega(n^3)$.

3. (15 points) A minimum bottleneck spanning tree (MBST) in an undirected connected weighted graph is a spanning tree in which the most expensive edge is as cheap as possible. Prove that a Minimum Spanning Tree (MST) is necessarily an MBST, and that an MBST is not necessarily a MST.

4. (15 points) A *maximum* matching in a graph $G$ is a matching of largest size. A *maximal* match is a matching where the addition of any other edge violates the matching constraint. A maximal matching does not need to be a maximum matching. However, a maximum matching is indeed a maximal matching.

   Prove that if $G$ is a graph with a maximum matching of size $2k$, the smallest maximal matching it could contain is of size $k$.