MLlib and Distributing the Singular Value Decomposition

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DATABRICKS

Spark

Outline

Example Invocations

Benefits of Iterations

Singular Value Decomposition

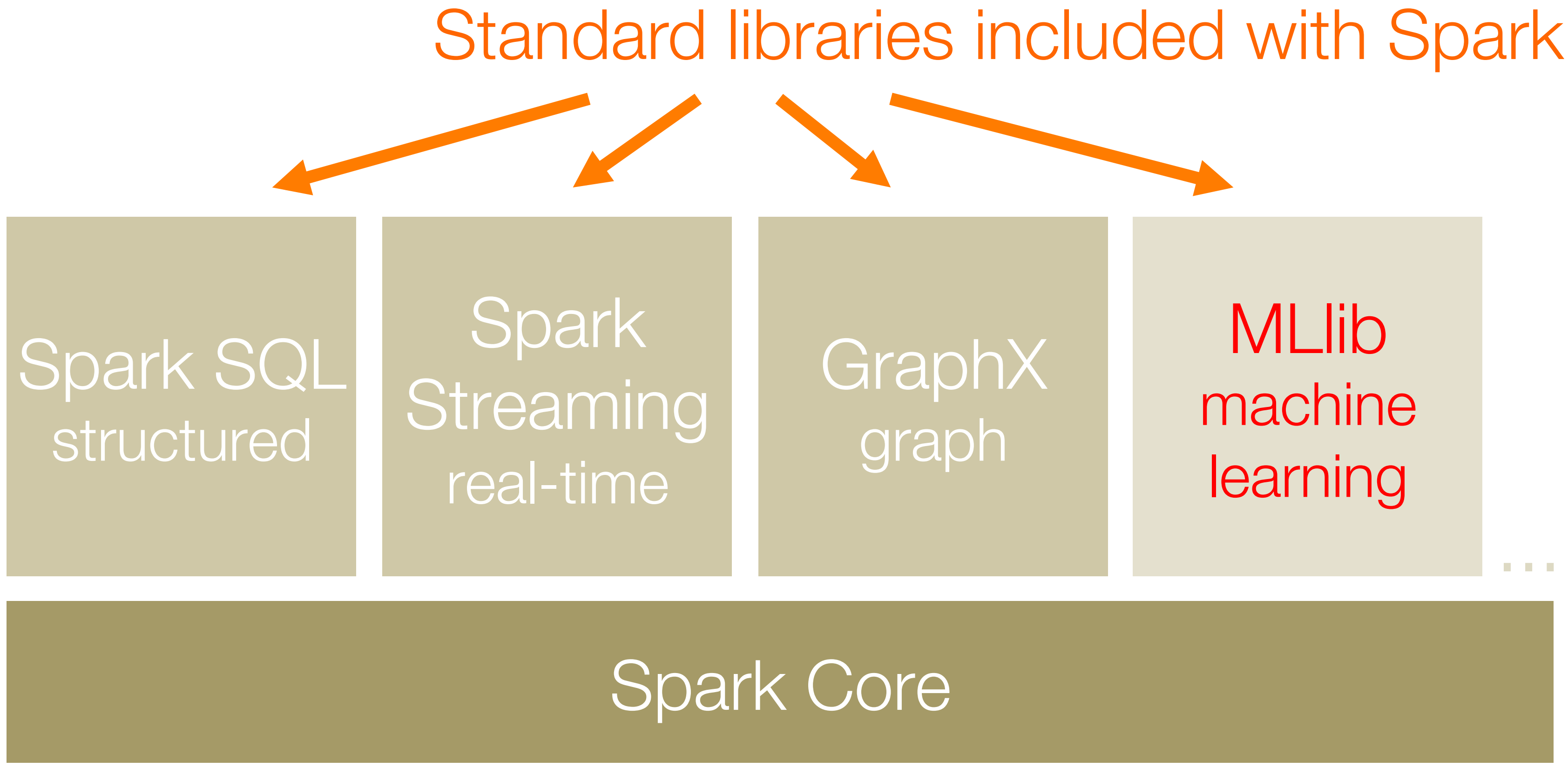
All-pairs Similarity Computation

MLlib + {Streaming, GraphX, SQL}

Future Directions

Introduction

A General Platform



MLlib History

MLlib is a Spark subproject providing machine learning primitives

Initial contribution from AMPLab, UC Berkeley

Shipped with Spark since Sept 2013

MLlib: Available algorithms

classification: logistic regression, linear SVM, naïve Bayes, least squares, classification tree

regression: generalized linear models (GLMs), regression tree

collaborative filtering: alternating least squares (ALS), non-negative matrix factorization (NMF)

clustering: k-means||

decomposition: SVD, PCA

optimization: stochastic gradient descent, L-BFGS

Example Invocations

Example: K-means

```
// Load and parse the data.
val data = sc.textFile("kmeans_data.txt")
val parsedData = data.map(_.split(‘ ’).map(_.toDouble)).cache()

// Cluster the data into two classes using KMeans.
val clusters = KMeans.train(parsedData, 2, numIterations = 20)

// Compute the sum of squared errors.
val cost = clusters.computeCost(parsedData)
println("Sum of squared errors = " + cost)
```

Example: PCA

```
// compute principal components
val points: RDD[Vector] = ...
val mat = RowRDDMatrix(points)
val pc = mat.computePrincipalComponents(20)

// project points to a low-dimensional space
val projected = mat.multiply(pc).rows

// train a k-means model on the projected data
val model = KMeans.train(projected, 10)
```

Example: ALS

```
// Load and parse the data
val data = sc.textFile("mllib/data/als/test.data")
val ratings = data.map(_.split(',') match {
    case Array(user, item, rate) =>
      Rating(user.toInt, item.toInt, rate.toDouble)
})

// Build the recommendation model using ALS
val model = ALS.train(ratings, 1, 20, 0.01)

// Evaluate the model on rating data
val usersProducts = ratings.map { case Rating(user, product, rate) =>
  (user, product)
}
val predictions = model.predict(usersProducts)
```

Benefits of fast iterations

Optimization

At least two large classes of optimization problems humans can solve:

- Convex Programs
- Singular Value Decomposition

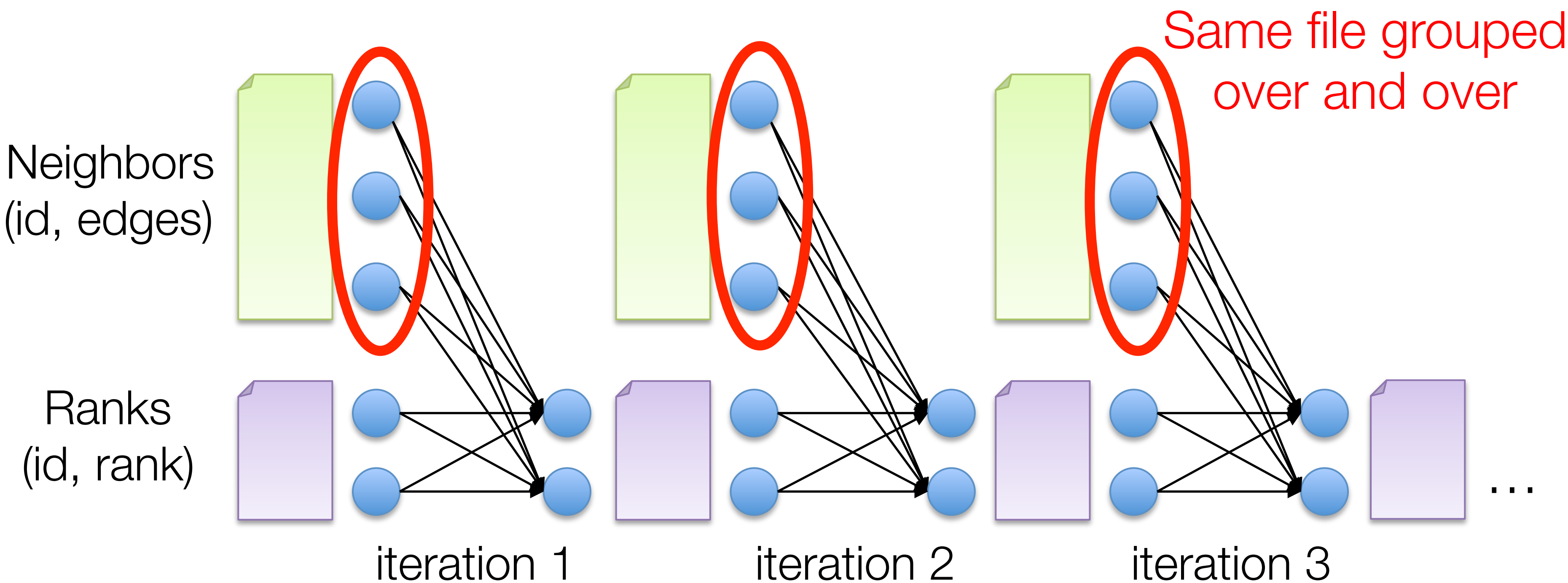
Optimization - LR

```
data = spark.textFile(...).map(readPoint).cache()

w = numpy.random.rand(D)

for i in range(iterations):
    gradient = data.map(lambda p:
        (1 / (1 + exp(-p.y * w.dot(p.x)))) * p.y * p.x
    ).reduce(lambda a, b: a + b)
    w -= gradient

print “Final w: %s” % w
```

MR PageRank

Repeatedly multiply sparse matrix and vector

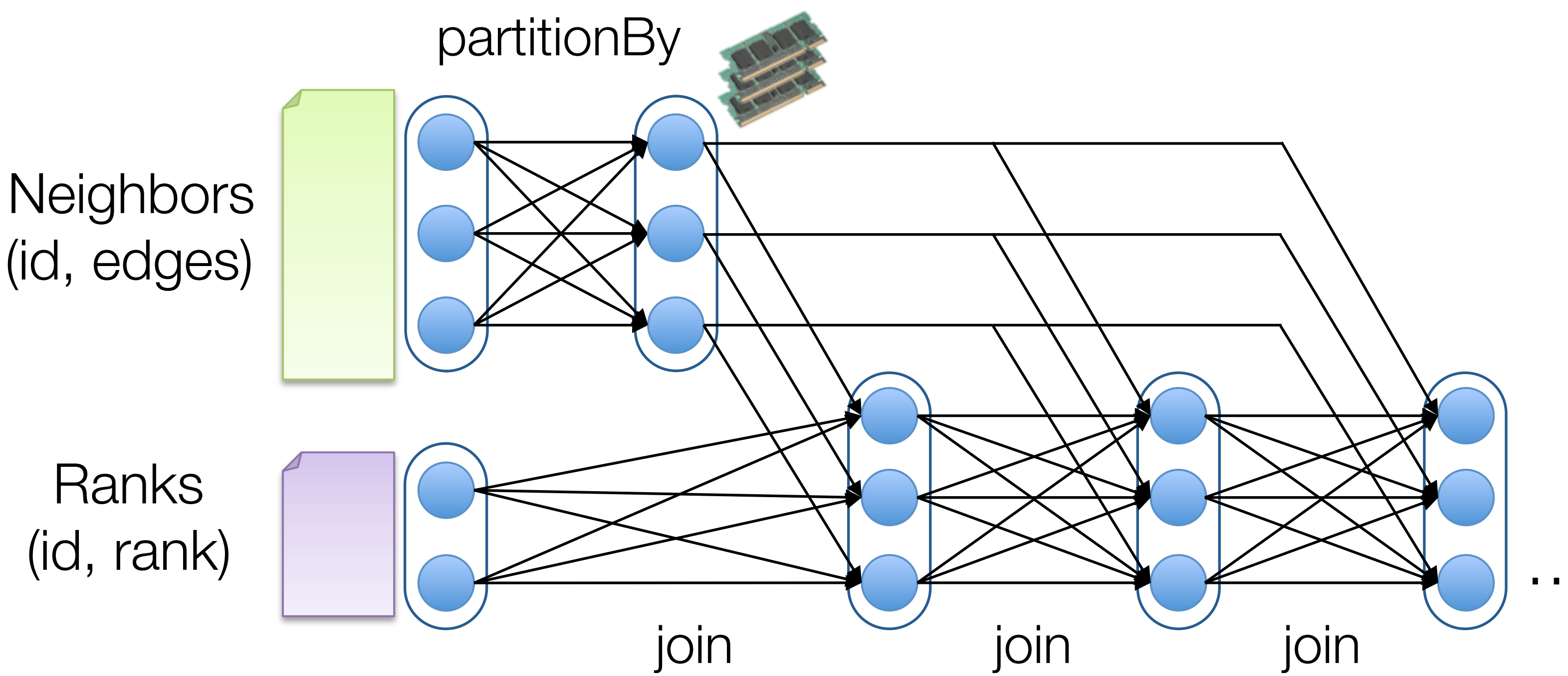
Requires repeatedly hashing together page adjacency lists and rank vector



Spark PageRank

Using cache(), keep neighbor lists in RAM

Using partitioning, avoid repeated hashing



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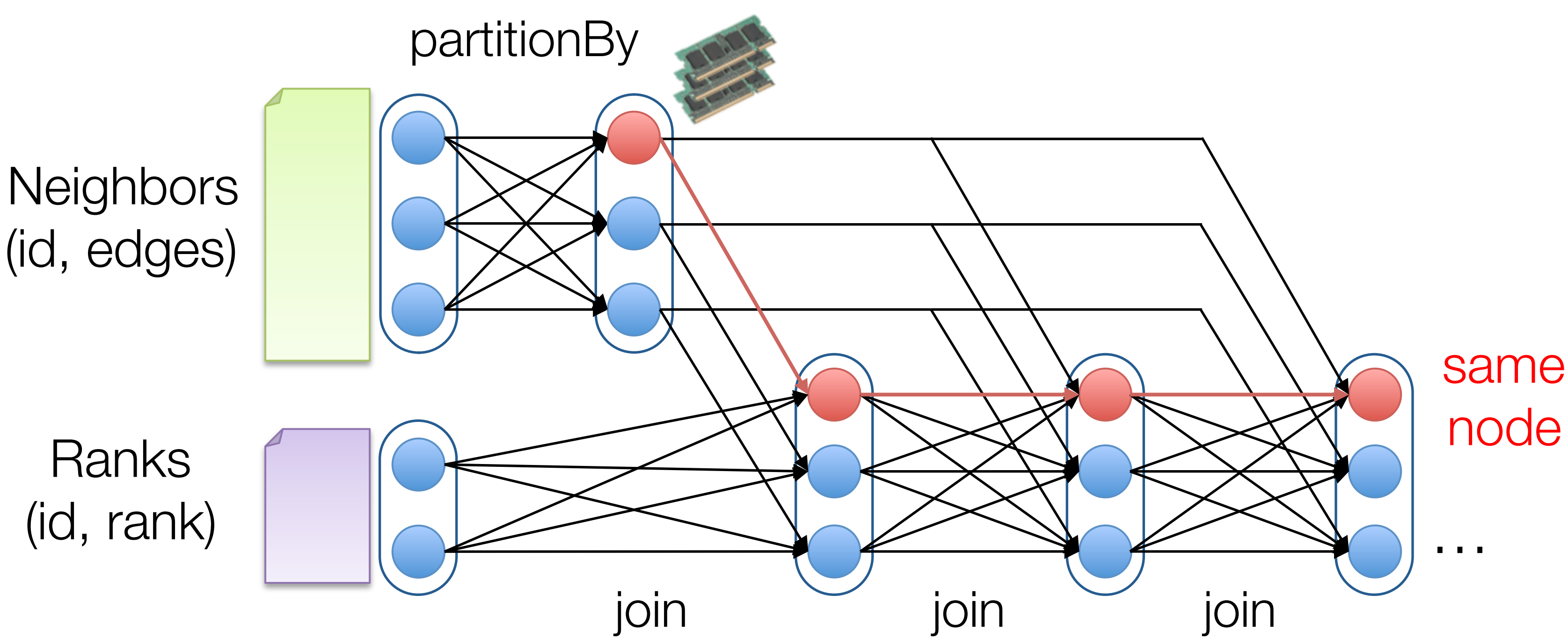

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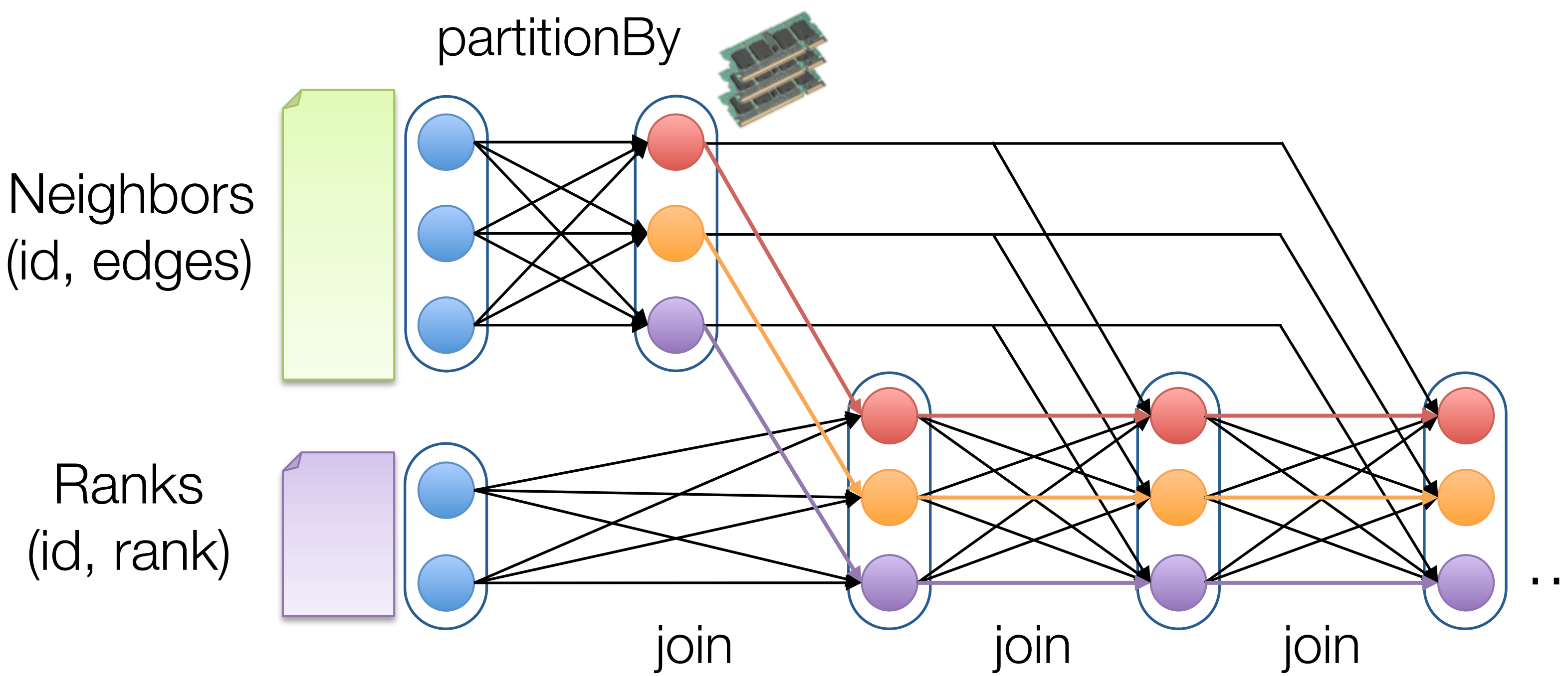


PageRank Code

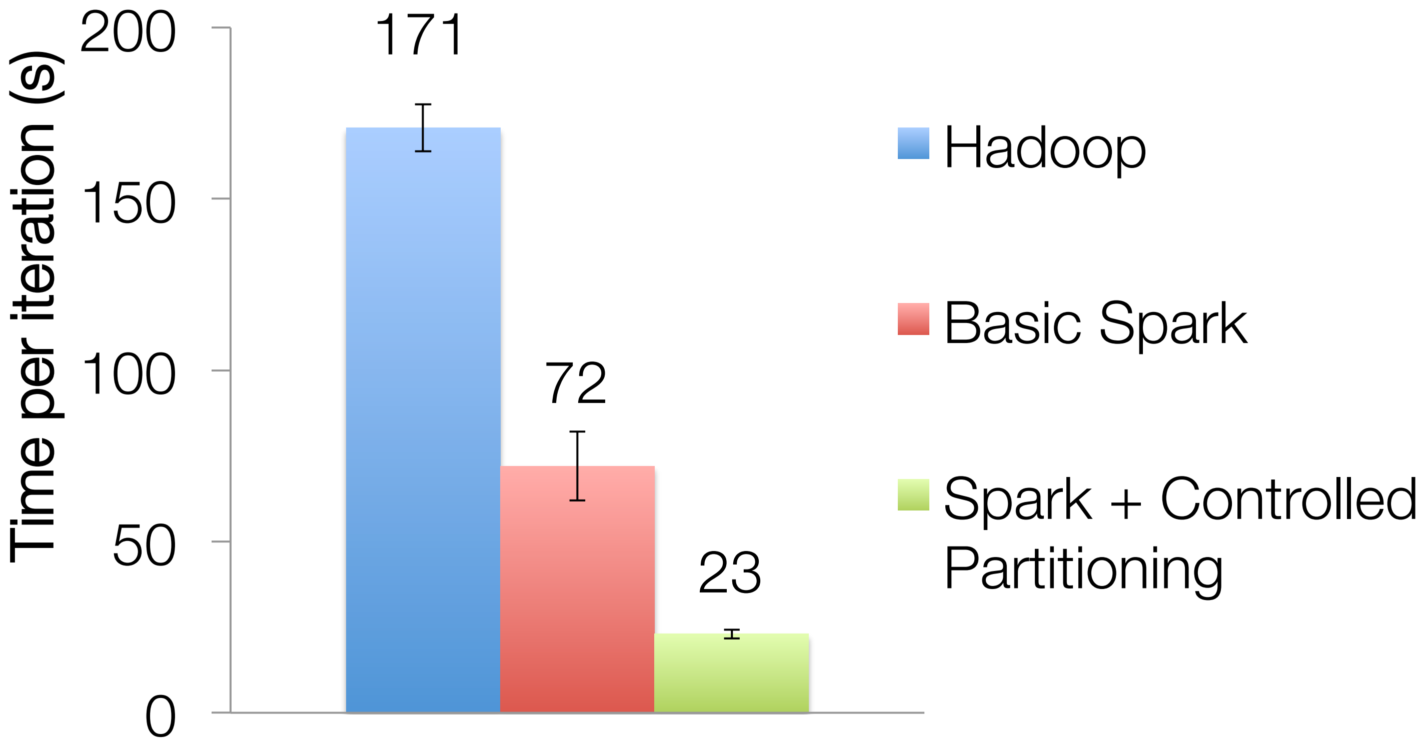
```
# RDD of (id, neighbors) pairs
links = spark.textFile(...).map(parsePage)
            .partitionBy(128).cache()

ranks = links.mapValues(lambda v: 1.0)  # RDD of (id, rank)

for i in range(ITERATIONS):
    ranks = links.join(ranks).flatMap(
        lambda (id, (links, rank)):
            [(d, rank/links.size) for d in links]
    ).reduceByKey(lambda a, b: a + b)
```

Generalizes to Matrix Multiplication, opening many algorithms from Numerical Linear Algebra

PageRank Results



Deep Dive: Singular Value Decomposition

Singular Value Decomposition

Two cases: Tall and Skinny vs roughly Square

computeSVD function takes care of which one to call, so you don't have to.

SVD selection

```
if (n < 100 || k > n / 2) {
  // If n is small or k is large compared with n, we better compute the Gramian matrix first
  // and then compute its eigenvalues locally, instead of making multiple passes.
  if (k < n / 3) {
    SVDMode.LocalARPACK
  } else {
    SVDMode.LocalLAPACK
  }
} else {
  // If k is small compared with n, we use ARPACK with distributed multiplication.
  SVDMode.DistARPACK
}
```

Tall and Skinny SVD

- Given $m \times n$ matrix A, with $m \gg n$.
- We compute $A^T A$.
- $A^T A$ is $n \times n$, considerably smaller than A.
- $A^T A$ is dense.
- Holds dot products between all pairs of columns of A.

$$A = U\Sigma V^T \qquad A^T A = V\Sigma^2 V^T$$

Square SVD via ARPACK

Very mature Fortran77 package for computing eigenvalue decompositions

$$K_n = \begin{bmatrix} b & Ab & A^2b & \cdots & A^{n-1}b \end{bmatrix}$$

JNI interface available via netlib-java

Distributed using Spark distributed matrix-vector multiplies!

Deep Dive: All pairs Similarity

Deep Dive: All pairs Similarity

Compute via DIMSUM: “Dimension Independent Similarity Computation using MapReduce”

Will be in Spark 1.2 as a method in RowMatrix

All-pairs similarity computation

- Given $m \times n$ matrix A, with $m \gg n$.

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

- A is tall and skinny, example values $m = 10^{12}, n = 10^6$
- A has sparse *rows*, each row has at most L nonzeros.
- A is stored across hundreds of machines and cannot be streamed through a single machine.

Naïve Approach

Algorithm 1 NaiveMapper(r_i)

for all pairs (a_{ij}, a_{ik}) in r_i **do**
  Emit $((j, k) \to a_{ij} a_{ik})$
end for

Algorithm 2 NaiveReducer($((i, j), \langle v_1, \ldots, v_R \rangle)$)

output $c_i^T c_j \to \sum_{i=1}^{R} v_i$

Naïve approach: analysis

- Very easy analysis
- 1) Shuffle size: $O(mL^2)$
- 2) Largest reduce-key: $O(m)$
- Both depend on m, the larger dimension, and are intractable for $m = 10^{12}, L = 100$.
- We'll bring both down via clever sampling
- Assuming column norms are known or estimates available

DIMSUM Sampling

Algorithm 3 DIMSUMv2Mapper(r_i)

for all a_{ij} in r_i **do**
  With probability $\min\left(1, \frac{\sqrt{\gamma}}{||c_j||}\right)$
  for all a_{ik} in r_i **do**
    With probability $\min\left(1, \frac{\sqrt{\gamma}}{||c_k||}\right)$
    emit $((j,k) \rightarrow \frac{a_{ij}a_{ik}}{\min(\sqrt{\gamma},||c_j||)\min(\sqrt{\gamma},||c_k||)})$
  end for
end for

DIMSUM Analysis

The algorithm outputs b_{ij}, which is a matrix of cosine similarities, call it B.

Four things to prove:

1. Shuffle size: $O(nL\gamma)$
2. Largest reduce-key: $O(\gamma)$
3. The sampling scheme preserves similarities when $\gamma = \Omega(\log(n)/s)$
4. The sampling scheme preserves singular values when $\gamma = \Omega(n/\epsilon^2)$

DIMSUM Proof

Theorem

For any two columns c_i and c_j having $\cos(c_i, c_j) \geq s$, let B be the output of DIMSUM with entries $b_{ij} = \frac{1}{\gamma}\sum_{k=1}^{m} X_{ijk}$ with X_{ijk} as the indicator for the k'th coin in the call to DIMSUMMapper. Now if $\gamma = \Omega(\alpha/s)$, then we have,

$$\Pr\left[||c_i||||c_j||b_{ij} > (1+\delta)[A^T A]_{ij}\right] \leq \left(\frac{e^\delta}{(1+\delta)^{(1+\delta)}}\right)^\alpha$$

and

$$\Pr\left[||c_i||||c_j||b_{i,j} < (1-\delta)[A^T A]_{ij}\right] < \exp(-\alpha\delta^2/2)$$

Relative error guaranteed to be low with high probability.

Spark implementation

Magnitudes shipped with every task

Makes life much easier than e.g. MapReduce

Ongoing Work in MLlib

multiclass decision trees

stats library (e.g. stratified sampling, ScaRSR)

ADMM

LDA

All-pairs similarity (DIMSUM)

General Convex Optimization

MLlib + {Streaming, GraphX, SQL}

MLlib + Streaming

As of Spark 1.1, you can train linear models in a streaming fashion

Model weights are updated via SGD, thus amenable to streaming

More work needed for decision trees

MLlib + SQL

```
points = context.sql(“select latitude, longitude from tweets”)
model = KMeans.train(points, 10)
```

MLlib + GraphX

```
// assemble link graph
val graph = Graph(pages, links)
val pageRank: RDD[(Long, Double)] = graph.staticPageRank(10).vertices

// load page labels (spam or not) and content features
val labelAndFeatures: RDD[(Long, (Double, Seq((Int, Double)))] = ...
val training: RDD[LabeledPoint] =
  labelAndFeatures.join(pageRank).map {
    case (id, ((label, features), pageRank)) =>
      LabeledPoint(label, Vectors.sparse(features ++ (1000, pageRank))
}

// train a spam detector using logistic regression
val model = LogisticRegressionWithSGD.train(training)
```

Future of MLlib

General Convex Optimization

Distribute CVX by backing CVXPY with PySpark

Easy-to-express distributable convex programs

Need to know less math to optimize complicated objectives

```
from cvxpy import *

# Create two scalar optimization variables.
x = Variable()
y = Variable()

# Create two constraints.
constraints = [x + y == 1,
               x - y >= 1]

# Form objective.
obj = Minimize(square(x - y))

# Form and solve problem.
prob = Problem(obj, constraints)
prob.solve()  # Returns the optimal value.
print "status:", prob.status
print "optimal value", prob.value
print "optimal var", x.value, y.value
```

```
status: optimal
optimal value 0.999999989323
optimal var 0.999999998248 1.75244914951e-09
```

Spark and ML

Spark has all its roots in research, so we hope to keep incorporating new ideas!

Next Speakers

Ameet: History of MLlib and the research on it at Berkeley

Ankur: Graph processing with GraphX

TD: Spark Streaming